Bayesian Inference and Model Comparison for Solar Atmospheric Seismology

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1. Solar Atmospheric Seismology

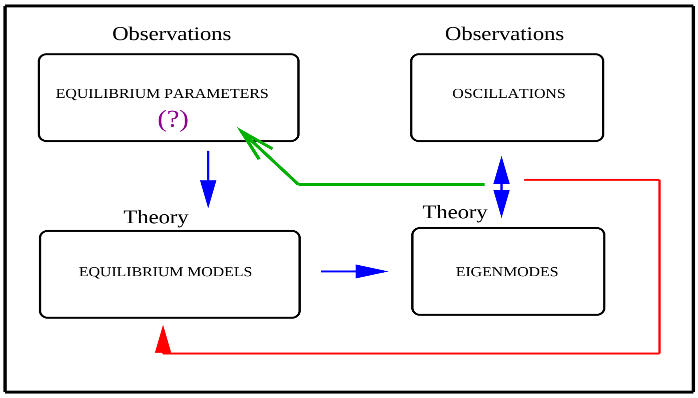
AIM: Determination of unknown physical parameters in the corona by comparison of observed and theoretical properties of waves and oscillations.

OBSERVATIONS: Damped transverse oscillations in magnetic waveguides. Fundamental kink mode and first overtone. Time and spatial damping.

THEORY: Damping by resonant absorption of MHD kink oscillations gives a good explanation for the observed rapid decay.

THIS WORK: We propose and use Bayesian inference and model comparison techniques for solar atmospheric seismology.

Systematic of MHD coronal seismology



2. Bayesian Analysis

$$p(\boldsymbol{\theta}|d) = \frac{p(d|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(d)}$$

BAYES' THEOREM:

$p(\boldsymbol{\theta}|d)$: posterior; $p(d|\boldsymbol{\theta})$: likelihood function; $p(\boldsymbol{\theta})$: prior; $p(d)$: evidence

State of knowledge on model parameters $\boldsymbol{\theta}$ is a combination of what is known a priori independently of the data, $p(\boldsymbol{\theta})$, and the likelihood of obtaining a data realization actually observed as a function of the parameter vector, $p(d|\boldsymbol{\theta})$.

PARAMETER INFERENCE:

How each parameter is constrained by data

$$p(\theta_i|d) = \int p(\boldsymbol{\theta}|d)d\theta_1 \ldots d\theta_{i-1}d\theta_{i+1}\ldots d\theta_N$$

MODEL COMPARISON:

Bayes factors: evidence of Model i against Model j, in view of data r

$$BF_{ij} = \frac{p(M_i|r)}{p(M_j|r)} = \frac{p(r|M_i)}{p(r|M_j)}\frac{p(M_i)}{p(M_j)}$$

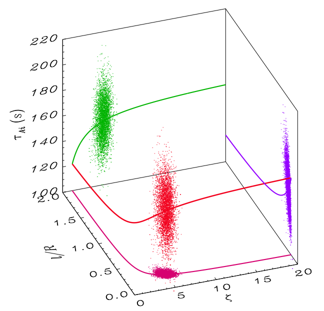
3. Coronal Loop Oscillations

Bayesian inversion for resonantly damped coronal loop oscillations

Observations: (P= 232 s , τ_d/P=3.6): period and damping rate

Unknowns: dens. contrast ζ, transverse scale l/R, Alfvén travel time τ_{Ai}

Arregui & Asensio Ramos (2011)



CLASSIC INVERSION

Infinite number of equally valid solutions (solid lines)

BAYESIAN INVERSION

The three unknowns can be constrained

Propagation of observational data uncertainties by means of marginal posteriors

τ_{Ai} =**152.4**$^{+15.4}_{-16.2}$ l/R= **0.26**$^{+0.02}_{-0.02}$

ζ= **4.99**$^{+0.52}_{-0.53}$

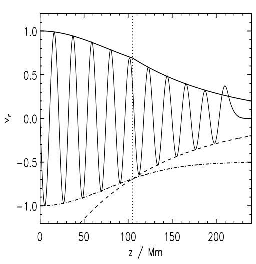
4. Propagating Kink Waves

Bayesian inversion of transverse density structuring

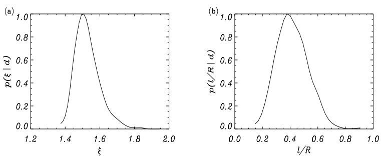
Observations: gaussian/exponential damping length scales

Unknowns: density contrast ζ, transverse scale l/R

Damping: Gaussian/exponential regimes (Pascoe et al. 2013)



MARGINAL POSTERIORS
Arregui, Asensio Ramos, Pascoe (2013)



Two damping regimes enable to constrain transverse density structuring

ζ=**1.52**$^{+0.08}_{-0.05}$, l/R=**0.41**$^{+0.12}_{-0.10}$

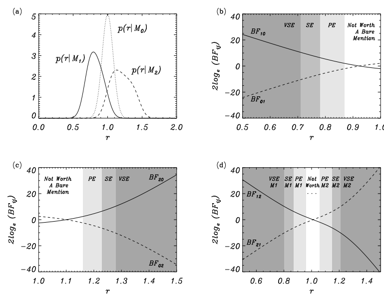
5. Multiple Harmonic Oscillations

Bayesian model comparison uniform vs. stratified vs. expanding tubes

Observations: multiple harmonic oscillations with period ratio r

Models: M0: uniform tube; M1: stratified tube; M2: expanding tube

Arregui, Asensio Ramos, Díaz (2013): Evidence of one model against other in view of data



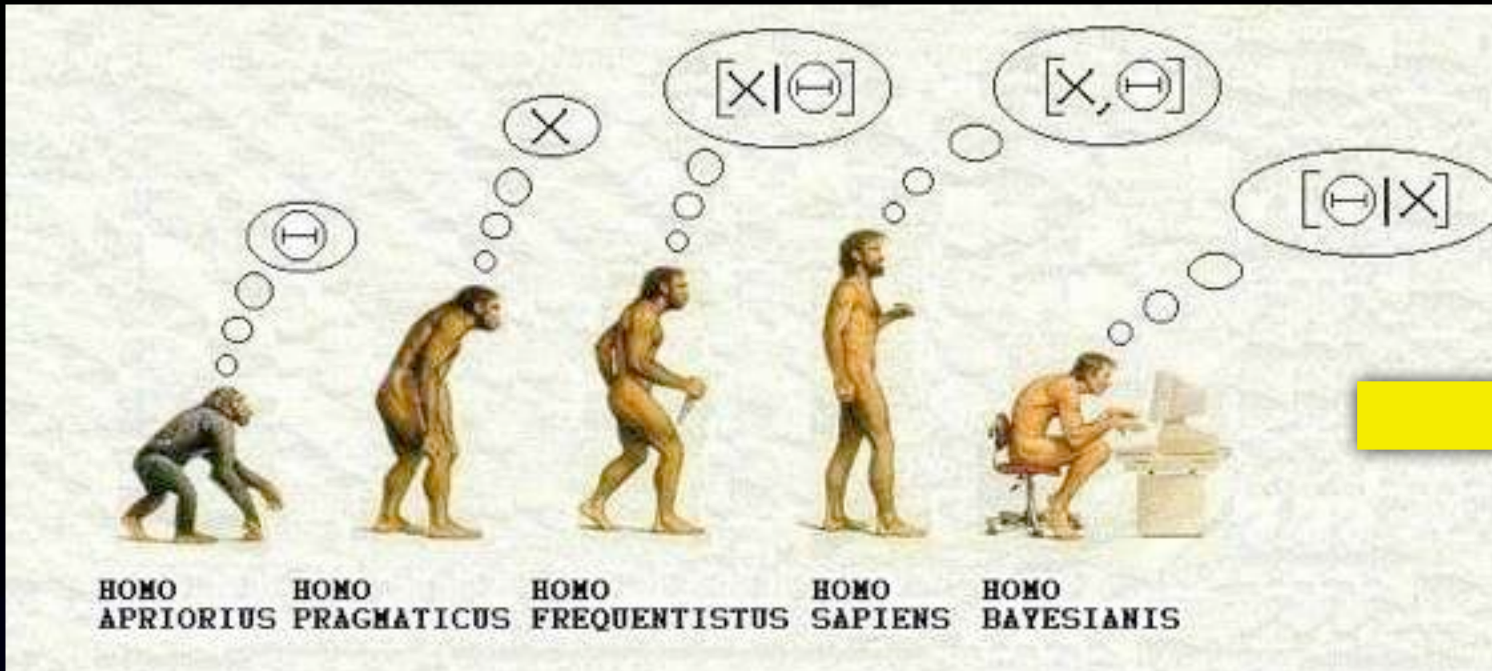
(a) Marginal Likelihoods
Uniform likely for $r \sim 1$
Stratified likely for $r < 1$
Expanding likely for $r > 1$

(b)-(d) Bayes Factors
Evidence for M1 vs. M0
Evidence for M2 vs. M0
Evidence for M1 vs. M2

Not worth ... more than a bare mention
PE: positive evidence; SE: strong evidence; VSE: very strong evidence

Conclusions

- ⋆ Bayesian analysis tools enable us to perform parameter inference and model comparison combining observations of transverse oscillations in the corona with MHD wave theory results.
- ⋆ Parameter inference successful in determining Alfvén speed and transverse density structuring in oscillating waveguides.
- ⋆ Method incorporates consistently calculated confidence levels and uncertainties.
- ⋆ Bayesian model comparison enables us to assess quantitatively which hypothesis, among competing mechanisms, better explains data.



Bayesian Seismology of the Solar Atmosphere

take the next step ... and be Bayesian!

pourquoi pas?

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